PERSPECTIVES

PHOENIX CENTER FOR ADVANCED LEGAL & ECONOMIC PUBLIC POLICY STUDIES

Estimating Betas in Practice: Alternatives that Matter and Those that Do Not

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Introduction

Pricing risk is essential to investment decisions. A commonly used financial model to quantify risk by investors, academic researchers, and policy analysts is the Capital Asset Pricing Model ("CAPM").[1] The key parameter of the CAPM is Beta (β), which is a measure of the systematic risk of a security or portfolio of securities in comparison to entire market. If Beta is greater (less than) unity, then the security or portfolio is more (less) volatile than the market.

Free access to historical stock price data has greatly expanded the opportunity for investment analysis and financial research by individual investors, students, policy analysts, and even academic researchers without access to expensive databases like that maintained by *The Center for Research in Security Prices* ("CRSP").[2] While Yahoo! Finance is unlikely to replace CRSP as *the* preferred source for quality financial data, research suggests that the free online data sources are sufficiently accurate for research and education.[3]

Estimating Beta is an especially common use of online financial data.[4] A number of "how to" guides explain the mechanics of estimating Beta (including access to formatted spreadsheets), but a review of these online materials points to a number of practical questions that require more attention than they are typically given.[5] First, a stock's "price" comes in multiple forms, with some (but not all) data services offering both a *Closing Price*, which is the actual observed price on the particular day of trading, and an *Adjusted Closing Price*, where the observed prices are adjusted for dividends. (Almost all data sources adjust for splits.) Second, academic studies often estimate Beta using excess returns, which equal the stock and index raw returns less a risk-free rate, while many "how to" guides and most financial services use raw returns. Using different measures of price and returns is certain to affect the size of Beta, but by how much?

> ***[T]he choice of using weekly or monthly returns and estimating two- or five-year Betas are more impactful decisions than is the accounting for dividends or the risk-free rate.***

In this PERSPECTIVE, I offer some evidence on these two questions in the most obvious of ways—I compute Beta using different prices and returns and compare them. I demonstrate that Betas are nearly identical whether or not the returns are adjusted for dividends and whether raw returns are used instead of excess returns. By far, the choice of using weekly or monthly returns and estimating two- or five-year Betas are more impactful decisions than is the accounting for dividends or the risk-free rate. That said, simpler, less data intensive estimates of Beta appear valid.

Background

Based on the CAPM, the expected risk premium of a security is

$$r_e - r_f = \beta(r_m - r_f) \tag{1}$$

where r_e is the security's return, r_m is the return on a broad market index (e.g., the S&P 500), r_f is the return on a risk-free asset (e.g., a Treasury Bill or Bond), and β measures how the volatility of the security compares to the volatility of a broad measure of security returns (or the market on whole). If the security is more volatile than the market ($\beta > 1$), then the required return on the security must be greater than the market return to compensate the investor for risk (and vice versa).

Beta is defined as

$$\beta = \frac{Cov\left(r_e - r_f, r_m - r_f\right)}{Var(r_m - r_f)}, \tag{2}$$

which is the ratio of the covariance of excess returns to the variance of the market excess returns. Equivalently, Beta may be estimated by a least-squares regression of the form,

$$r_e - r_f = \alpha + \beta\left(r_m - r_f\right) + \varepsilon, \tag{3}$$

where ε_t is the econometric disturbance.[6] Popular financial services (e.g., Value Line, Yahoo! Finance, and so forth) that report the Beta of individual company stocks and most "how to" guides for estimating Beta employ a simplified version of these expressions, estimating Beta using either,

$$\beta' = \frac{Cov\left(r_e, r_m\right)}{Var(r_m)}, \tag{4}$$

or the least-squares regression,

$$r_e = \alpha' + \beta' r_m + e, \tag{5}$$

where both use raw returns rather than excess returns thus reducing the data demands for estimation. Of course, the difference approaches imply that $\beta \neq \beta'$ and $\alpha \neq \alpha'$, though I am unaware of any analysis demonstrating how large these discrepancies are. The discrepancies are not simply related to the size of r_f but also are systematically related to the size of β.

Furthermore, there is the question of which price to use when calculating returns. Some data sources, including Yahoo! Finance, offer multiple closing prices, usually a *Closing Price* and an *Adjusted Closing Price*, the latter which is adjusted for dividends. Some data services offer only the *Closing Price*, such as eoddata.com.

The difference between the two prices in shown in Equation (6). If the price of the security is p, then the return from holding the stock between the two periods (r) is, generally,

$$r_e = \frac{p_t - p_{t-1} + d_t}{p_{t-1}}. \tag{6}$$

where d_t is a dividend paid in period t. Between most short intervals of time, d_t is zero, and for many stocks it is always zero (no dividend is paid). If d_t is zero, then the returns computed using the *Closing Price* and *Adjusted Closing Price* are the same. When d_t is positive, it affects the return, and, in doing so, will affect the estimate of β.[7] Dividends are infrequently paid, however. A quarterly dividend impacts 1.5% of daily returns, 7.6% of weekly returns, and 33% of monthly returns. The effect of adjusted prices on β, therefore, should be more impactful for monthly than daily data.

Equations (2,3) are theoretically correct but Equations (4,5) require less data and fewer calculations, and the financial services typically report estimates from the latter. And, depending on the preferred data source, the analysts may be limited to using data on either *Closing Price* and

Adjusted Closing Price. Thus, the size of the difference between the two methods and closing price types are interesting and practically-relevant questions. Moreover, Betas may be estimated using daily, weekly, or monthly returns, and data typically spans a range of one-year to up to six-years of data—choices sure to impact the size of the estimated Beta. Next, some simple empirical tests are performed to assess the impact on Beta of these choices.

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Empirical Method

In an effort to assess the impact of these different methods, prices, and time spans, data are collected from Yahoo! Finance on the *Closing Price* and *Adjusted Closing Price* for ten dividend-paying and ten non-dividend paying stocks for the five-year period 2013 through 2017.[8] Data are also obtained on the S&P 500 index (^GSPC) and the risk-free rate is measured by Yahoo! Finance's 13-week Treasury Bill yield (^IRX). This choice of risk-free rate is desirable in that it is easily obtained from Yahoo! Finance and thus available to analysts.

The Risk-Free Rate

Unlike security prices, the return cannot be calculated as a simple percentage difference between two prices. The risk-free rate is measured as an annual yield, so it must be converted to the desired frequency. The following procedure is used.

Let y_t be the yield reported in ^IRX (divided by 100) on a Treasury bill maturing in three months (or a quarter year). For the three-month bill, the yield is a discount on the face value and no interest payments are made. Thus, for a $100 face value, if the annual yield is reported to be 1%, then the bond sold at a price of $99.75 and pays $100 three months later. So, to convert the annual yield to the frequency of interest (q), the following formula is used:

$$r_{f,t}^{q} = \left[1 + \left(\frac{1}{(1-0.25y_{t-1})} - 1\right)\right]^{1/q} - 1, \quad (7)$$

where the term in parenthesis is the 3-month return on the treasury bill and q is 3 for monthly data, 12 for weekly data, and 90 for daily data.

The Estimated Betas

The goal is to compare the β estimated using the *Closing Price* and *Adjusted Closing Price* and likewise to compare the β estimated using raw or excess returns. Thus, four β are calculated and compared for each stock. The notation is β_{ax}, where *a* is C for *Closing Price* and A for *Adjusted Closing Price*, and *x* is R for raw returns and E for excess returns. Betas are computed using weekly or monthly data for the five-year window, and weekly data for a two-year window.

My interest is in material differences in β due to the choices made by the analyst, so I report the β rounded to the second decimal place, which is somewhat standard convention for reporting β. All the β are statistically significant using reported errors, so I do not report the standard errors. Detailed results are available upon request.

Dividend Paying Stocks

To begin, the β are estimated for ten dividend-paying stocks using weekly data over a five-year window.[9] For dividend paying stocks there will be a difference in the returns computed using the *Closing Price* and *Adjusted Closing Price*. That said, dividends are paid infrequently, and a quarterly payment schedule will only impact 20 of the 260 returns in the five-year sample of weekly data. The difference between the returns calculated from the two prices should be small. Subtracting a relatively small risk-free return from the equity returns is also expected to have a small effect on the estimated β.

Table 1. Dividend Stocks β
Five-Year, Weekly (N= 260)

Symbol	β_{AR}	β_{CR}	β_{AE}	β_{CE}
AGM	1.16	1.16	1.16	1.16
AVGO	1.27	1.27	1.26	1.26
BXS	1.30	1.30	1.30	1.30
CATY	1.32	1.32	1.32	1.32
FFG	0.98	0.98	0.96	0.96
MTN	0.80	0.80	0.79	0.79
RCL	1.44	1.45	1.44	1.44
RF	1.27	1.27	1.27	1.27
STI	1.28	1.28	1.28	1.28
VLO	1.28	1.28	1.27	1.27

Results are summarized in Table 1. The differences across the data treatments are trivially small and mostly undetectable at the second decimal place. The largest difference is a change from 0.98 to 0.96, and this difference is attributable to the use of raw and excess returns. Which closing price is used has almost no effect.

Betas are nearly identical whether or not the returns are adjusted for dividends and whether raw returns are used instead of excess returns.

Table 2 summarizes the results for β estimated using monthly data, where the dividend payments may have a larger effect on returns. While the frequency change leads to βs that are different than those reported in Table 1, the differences across the data choices continue to be very small. The largest difference is 0.88 to 0.84, reflecting adjustments for dividends. Again, using raw instead of excess returns has almost no effect on β.

Table 2. Dividend Stocks β
Five-Year, Monthly (N= 60)

Symbol	β_{AR}	β_{CR}	β_{AE}	β_{CE}
AGM	1.26	1.26	1.26	1.25
AVGO	0.90	0.91	0.90	0.90
BXS	1.36	1.36	1.36	1.36
CATY	1.35	1.35	1.35	1.36
FFG	0.88	0.88	0.84	0.84
MTN	0.42	0.42	0.42	0.42
RCL	1.04	1.04	1.04	1.04
RF	1.33	1.33	1.33	1.33
STI	1.34	1.34	1.35	1.35
VLO	1.14	1.14	1.15	1.15

Comparing Tables 1 and 2, it is clear to see that the choice of using weekly or monthly data is far more significant in terms of the size of the β than is the adjustment for dividends or the risk-free return. The less data-intensive methods and data unadjusted for dividends provide good estimates of β.

Table 3. Dividend Stocks β
Two-Year, Weekly (N= 104)

Symbol	β_{AR}	β_{CR}	β_{AE}	β_{CE}
AGM	1.74	1.74	1.74	1.74
AVGO	1.69	1.69	1.69	1.69
BXS	1.83	1.83	1.82	1.82
CATY	1.76	1.76	1.74	1.74
FFG	1.48	1.48	1.47	1.47
MTN	0.99	0.99	1.01	1.01
RCL	1.66	1.66	1.65	1.65
RF	1.69	1.69	1.68	1.68
STI	1.56	1.56	1.54	1.54
VLO	1.05	1.05	1.05	1.05

In Table 3, a two-year β estimated with weekly data is summarized. Again, there are some material differences in the sizes of β between Table 3 and Tables 1 and 2, but these differences reflect only changes in frequency and time spans. Adjustments to the returns for dividends and the risk-free rate do not materially impact the estimate of β.

Non-Dividend Paying Stocks

For non-dividend paying stocks, there is no reason to expect a difference in the estimated β across the two types of closing prices.[10] The results above suggest accounting for the risk-free rate is also unimportant.

Table 4. Non-Dividend Stocks β
Five-Year, Weekly (N= 260)

Symbol	β_{AR}	β_{CR}	β_{AE}	β_{CE}
AMZN	1.19	1.19	1.19	1.19
BSX	0.90	0.90	0.90	0.90
CNC	0.99	0.99	0.99	0.99
EW	0.78	0.78	0.78	0.78
HSIC	0.88	0.88	0.88	0.88
IT	0.78	0.78	0.78	0.78
MHK	1.16	1.16	1.16	1.16
REGN	1.47	1.47	1.47	1.47
ULTA	0.82	0.82	0.82	0.82
VRSN	0.93	0.93	0.93	0.93

Results for the non-dividend paying stocks are summarized in Table 4. All the estimated β are identical, at least at two-decimal places.

Conclusion

In this PERSPECTIVE, I demonstrate that Betas are nearly identical whether or not the returns are adjusted for dividends and whether raw returns are used instead of excess returns. By far, the choice of using weekly or monthly returns or estimating two- or five-year Betas are more impactful than is the accounting for dividends or the risk-free rate. That said, simpler, less data intensive estimates of Beta appear valid. At a finer level of precision, the Betas will differ, but quantifying the systematic determinants of such differences are left for later research.

NOTES:

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1 For an overview of the CAPM, *see* W. Kenton, Capital *Asset Pricing Model-CAPM,* INVESTOPEDIA (December 7, 2018) (available at: https://www.investopedia.com/terms/c/capm.asp). For applications to public policy, *see*, *e.g*., R.A. Morin, NEW REGULATORY FINANCE (2006); I. Karafiath, *Using Dummy Variables in the Event Methodology*, 23 THE FINANCIAL REVIEW 351–7 (1988); M. Cichello and D.J. Lamdin, *Event Studies and the Analysis of Antitrust*, 13 INTERNATIONAL JOURNAL OF THE ECONOMICS OF BUSINESS 229-245 (2007) (available at: https://www.tandfonline.com/doi/abs/10.1080/13571510600784557?mobileUi=0&journalCode=cijb20); G.S. Ford & A.D. Kline, *Event Studies for Merger Analysis: An Evaluation of the Effects of Non-Normality on Hypothesis Testing*, in ANTITRUST POLICY ISSUES (P. Mariati, ed.) (2006) at Ch. 8. A draft is available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=925953); R.B. Ekelund, Jr., and G.S. Ford, *Innovation, Investment, and Unbundling: An Empirical Update*, 20 YALE JOURNAL ON REGULATION 383-388 (2003).

2 Center for Research in Security Prices (http://www.crsp.com).

3 F. Flanegin, S. Racic, and D. Rudd, *Accuracy and Cost of U.S. Financial Data*, 25 JOURNAL OF APPLIED BUSINESS RESEARCH 47-54 (2009) (available at: https://clutejournals.com/index.php/JABR/article/download/994/978).

4 *See*, *e.g*., http://finance.yahoo.com; https://www.nasdaq.com; https://stooq.com; https://www.wikihow.com/Calculate-Beta; https://www.investopedia.com/articles/investing/102115/what-beta-and-how-calculate-beta-excel.asp. STATA and other statistical software packages can download stock price data directly from a variety of sources included Yahoo! Finance, Google Finance, Quandl, among others.

5 A detailed "how to" guide is provided at: https://studylib.net/doc/8191578/lecture-14--implementing-capm.

6 The distribution of the residual is the subject of some debate. *See*, *e.g*., E.F. Fama and J.D. MacBeth, *Risk, Return, and Equilibrium: Empirical Tests*, 81 JOURNAL OF POLITICAL ECONOMY 607–636 (1973); G.S. Ford and A.D. Kline, *Event Studies for Merger Analysis: An Evaluation of the Effects of Non-Normality on Hypothesis Testing*, in P. Moriati, ed., ANTITRUST POLICY ISSUES 135–56 (Ch. 8) (2006) (draft available at: https://ssrn.com/abstract=925953).

7 Say, the closing price on Tuesday is $100, a dividend is paid on Wednesday equal to $2, and the closing price on Wednesday is $100.1. The *Closing Price* data will list the prices as $100 and $100.1, as observed, and the daily return is 0.1%, a small increase. The *Adjusted Closing Price*, alternately, lists the prices as $98 [= 100-2] and $100.1, and *all* prices prior to that Wednesday will be shifted down by the same $2 difference. The daily return is now as 2.1%, a large increase. The dividend is not really a surprise, however, and the market did not signal a much greater interest in the stock that day, despite the fact the adjusted prices suggest that is so. Over long spans of history, the *Closing Price* and *Adjusted Closing Price* will differ by large amounts for dividend-paying stocks, so if observed prices is the item of interest, then *Closing Price* should be used.

8 I also include the final month of 2012 so s to have a full five-years of data after the returns are computed. All analysis is conducted using Stata 15 and stock returns are obtained using -getsymbols-.

9 The stocks are chosen from the list provided at: https://money.usnews.com/investing/stocks/dividend-growers.

10 The stocks are chosen from the list provided at: http://www.dividend.com/investor-resources/sp-500-companies-that-dont-pay-dividends.